

RSA Exercises

RSA and Public Key Codes¹

1. Find the decoding key d for the code whose published values of N and e are $N = 233\,570\,063$, $e = 125$.

[$d = 136\,387\,061$]

2. Decrypt, by hand, the following RSA-encoded message ($N = 7387$, $e = 1357$):

2133 429 1126

You may assume the following was used to equate letter pairs with numbers:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36

[“NUMBER”]

3. Presume that a plaintext message is converted to a number by making the following substitutions: blank = 99, A=10, B=11, ..., Z=35. Decode the following message. It is a quotation from Shakespeare's "Hamlet". The coded message consists of four integers:

$$c_1 = 39\,25736\,57380\,83976$$

$$c_2 = 8\,66571\,70599\,56870$$

$$c_3 = 14569\,39934\,49451$$

$$c_4 = 14\,57541\,36754\,04137$$

The public key used for encryption was ($N = 59\,11142\,11035\,79513$, $e = 123$).

RSA Encoding²

4. Suppose you want to send a message to a friend. You look up your friend's RSA public-key and find that $N = 1964556481$ and $e = 456899$. In blocks of 4 alphabet letters using the ASCII code numerical alphabet assignment, send to your friend the message “THIS IS A SECRET”.

¹ The Oxford Math Center, <http://www.oxfordmathcenter.com/drupal7/node/206>

² <http://www.radford.edu/~npsigmon/courses/cryptography/RSAexercisesSection4.4.doc>

Solutions

1. $N = 233570063 \Rightarrow p = 14897 \quad q = 15679 \quad \varphi(N) = 233539488$

$e = 125 \Rightarrow d = 1/e \bmod \varphi(N) = 136387061$

2. $N = 7387 \Rightarrow p = 83 \quad q = 89 \quad \varphi(N) = 7216$

$e = 1357 \Rightarrow d = 1/e \bmod \varphi(N) = 5525$

$m_1 = c_1^d \bmod N = 2133^{5525} \bmod 7387 = 2431 \quad 24 = \text{"N"} \quad 31 = \text{"U"}$

$m_2 = c_2^d \bmod N = 429^{5525} \bmod 7387 = 2312 \quad 23 = \text{"M"} \quad 12 = \text{"B"}$

$m_3 = c_3^d \bmod N = 1126^{5525} \bmod 7387 = 1528 \quad 15 = \text{"E"} \quad 28 = \text{"R"} \quad \text{NUMBER}$